P.G. Sem - II, Paper - CC IX

Topology

Ques → Let N be the set of all positive integers and let J be the family consisting of the empty set ϕ and all subsets of N of the form $\{m, m+1, m+2, \ldots\}$ where $m \in N$. Show that J is a topology on N.

Ans. Let us consider

$$G_m = \{m, m+1, m+2, \ldots\ldots\}$$

Now $\phi \in J$

Also, $G_1 = \{1, 2, 3, \ldots\}$

$$= N \in J$$

Hence condition $[O_1]$ is satisfied.

Let $G_a \in J$ for every $a \in I$ where $I \subseteq N$

The set I contains a smallest +ve integer n_0.

Then $\bigcup_{a \in I} G_a = \{n_0, n_0+1, n_0+2 \ldots\}$

$$= G_{n_0} \in J$$

Hence condition $[O_2]$ is also satisfied.

Finally, Let $G_m, G_n \in J$ for $m, n \in N$

Then $G_m \cap G_n = G_n$ or G_m according as $m < n$ or $m > n$.

Thus $G_m \cap G_n \in J$

Hence condition $[O_3]$ is also satisfied.

Therefore J is a topology on N.